

Exercise 21

For the following exercises, use synthetic division to find the quotient.

$$(2x^3 + 7x^2 - 13x - 3) \div (2x - 3)$$

Solution

Synthetic division only works if the divisor is of the form $x - k$. Multiply the numerator and denominator by $1/2$ to make it so.

$$\frac{2x^3 + 7x^2 - 13x - 3}{2x - 3} = \frac{2x^3 + 7x^2 - 13x - 3}{2x - 3} \times \frac{1/2}{1/2} = \frac{x^3 + \frac{7}{2}x^2 - \frac{13}{2}x - \frac{3}{2}}{x - \frac{3}{2}}$$

Solving $x - \frac{3}{2} = 0$ gives $x = \frac{3}{2}$; this is the number that goes on the top left. Write out all the coefficients of the dividend to the right.

$$\begin{array}{r|rrrr} \frac{3}{2} & 1 & \frac{7}{2} & -\frac{13}{2} & -\frac{3}{2} \\ \hline & & & & \end{array}$$

Bring down the leading coefficient.

$$\begin{array}{r|rrrr} \frac{3}{2} & 1 & \frac{7}{2} & -\frac{13}{2} & -\frac{3}{2} \\ \hline & 1 & & & \end{array}$$

Multiply the top left number by the number brought down and put the result under the second coefficient of the dividend.

$$\begin{array}{r|rrrr} \frac{3}{2} & 1 & \frac{7}{2} & -\frac{13}{2} & -\frac{3}{2} \\ \hline & 1 & 3 & & \\ & & \frac{3}{2} & & \end{array}$$

Add the numbers in the second column.

$$\begin{array}{r|l} \frac{3}{2} & 1 \quad \frac{7}{2} \quad -\frac{13}{2} \quad -\frac{3}{2} \\ \hline & \frac{3}{2} \\ & \frac{15}{2} \\ \hline & 1 \quad 5 \end{array}$$

Multiply this sum of the second column by the top left number and put it in the next column.

$$\begin{array}{r|l} \frac{3}{2} & 1 \quad \frac{7}{2} \quad -\frac{13}{2} \quad -\frac{3}{2} \\ \hline & \frac{3}{2} \quad \frac{15}{2} \\ & \frac{9}{2} \quad \frac{45}{2} \\ \hline & 1 \quad 5 \end{array}$$

Add the numbers in the third column.

$$\begin{array}{r|l} \frac{3}{2} & 1 \quad \frac{7}{2} \quad -\frac{13}{2} \quad -\frac{3}{2} \\ \hline & \frac{3}{2} \quad \frac{15}{2} \\ & \frac{9}{2} \quad \frac{45}{2} \\ \hline & 1 \quad 5 \quad 1 \end{array}$$

Multiply this sum of the third column by the top left number and put it in the next column.

$$\begin{array}{r|rrrr} \frac{3}{2} & 1 & \frac{7}{2} & -\frac{13}{2} & -\frac{3}{2} \\ & & \frac{3}{2} & \frac{15}{2} & \frac{3}{2} \\ \hline & 1 & 5 & 1 & \end{array}$$

Add the numbers in the fourth column.

$$\begin{array}{r|rrrr|r} \frac{3}{2} & 1 & \frac{7}{2} & -\frac{13}{2} & -\frac{3}{2} \\ & & \frac{3}{2} & \frac{15}{2} & \frac{3}{2} \\ \hline & 1 & 5 & 1 & 0 \end{array}$$

This final result is the remainder, and the numbers to the left are the coefficients of the quotient, which is $x^2 + 5x + 1$.

$$(2x^3 + 7x^2 - 13x - 3) \div (2x - 3) = x^2 + 5x + 1$$